DOCUMENT RESUME

ED 062 160 SE 013 606

TITLE	Committee on the Undergraduate Program in Mathematics Newsletter, Number 7.
INSTITUTION	Committee on the Undergraduate Program in Mathematics, Berkeley, Calif.
PUB DATE	Feb 72
NOTE	24p.
AVAILABLE FROM	CUPM Central Office, P. O. Box 1024, Berkeley, California 94701 (Free)
EDRS PRICE	MF-$0.65 HC-$3.29
DESCRIPTORS	*College Mathematics; Instruction; *Large Group Instruction; Mathematics Education; *Staff Orientation; Teacher Education; *Teaching Assistants; Televised Instruction
IDENTIFIERS	CUPM; Teaching Modules

ABSTRACT

Following a "Statement on Teaching Experience as Part of Ph.D. Programs," two orientation programs for teaching assistants are described. Five recent experiments in teaching mathematics to large numbers of undergraduates are also described, one using television and the remainder using modules. (MM)

COMMITTEE ON THE UNDERGRADUATE PROGRAM IN MATHEMATICS

CUPM NEWSLETTER

CUPM Central Office - Post Office Box 1024 - Berkeley, California 94701 - (415) 527-2363

EDITORS
Andrew Sterrett, Executive Director (Denison University)
Paul J. Knopp, Associate Director (University of Houston)
William E. Mastrocola, Associate Director (Colgate University)
Katherine B. Magann, Administrative Assistant

Number 7 February, 1972

NEW METHODS FOR TEACHING ELEMENTARY COURSES AND FOR THE ORIENTATION OF TEACHING ASSISTANTS

CONTENTS

INTRODUCTION

On May 1, 1971, the College Teacher Preparation Panel of the Committee on the Undergraduate Program in Mathematics (CUPM) sponsored a conference in Chicago on new approaches to the interlocking problems of teaching mathematics to large numbers of students and training teaching assistants in mathematics. This Newsletter makes available to a wider audience reports on some interesting new programs, most of them described at that conference. These reports by no means constitute an exhaustive catalogue of the programs of these kinds now under way in this country, but they suggest some possibilities which other mathematics departments may wish to consider as they review their own programs.

The Newsletter begins with a "Statement on Teaching Experience as Part of Ph.D. Programs," drafted by the Panel and informally endorsed at the conference. This statement is largely an expanded expression of a position taken in previous CUPM documents (A Beginning Graduate Program in Mathematics for Prospective Teachers of Undergraduates, 1969, and Qualifications for A College Faculty in Mathematics, 1967) and sets the stage for the subsequent specific reports on programs for teaching assistants at Rensselaer Polytechnic Institute and the University of Minnesota. This much of the present Newsletter may be regarded as a sequel to the CUPM Newsletter Number 3, "Teaching Assistants" (August, 1968), which reports on earlier efforts along the same line.

The remainder of the Newsletter deals with recent experiments in teaching mathematics to large numbers of students with the aid of television at the University of Maryland and with the use of modules at Massachusetts Institute of Technology, Ohio State University, University of California (San Diego), and Western Michigan University.

All of the reports in this Newsletter are being distributed simply for the information of the mathematical community, and their appearance here does not constitute endorsement by CUPM or by any of its panels. Each program was carefully planned by and for the reporting institution, and adaptation to other institutions will require thorough study.

There is one point which we would emphasize. In the evaluation of any new programs, whether mentioned here or not, cost is inevitably a consideration. It should not, however, be the primary consideration: every instructional program should be judged above all on its educational effectiveness. A penny saved by unnecessarily sacrificing instructional quality is not a penny earned; it is a penny stolen.

STATEMENT ON TEACHING EXPERIENCE AS PART OF Ph.D. PROGRAMS

In most American universities today, a very large share of the teaching of elementary mathematics is done by graduate students.* Although their teaching responsibilities require perhaps 25 to 30 per cent of these students' time, they usually receive no academic credit for this work. In many cases their work is supervised only superficially and in some cases not at all. Because of the pressure exerted on teaching assistants to do well in their graduate courses and because of the lack of pressure to do well in their teaching, many of them understandably devote much less time than they should to their teaching responsibilities. Even those who are anxious to do a good job of teaching and are willing to spend the required time sometimes need the guidance of experienced teachers. The lack of provision for the supervision of teaching assistants thus adversely affects the quality of mathematics instruction at the university.

Nearly all of the Ph.D. graduates in mathematics take academic positions, and even those who do not must still "teach" in the sense that they must explain mathematical ideas to others. Thus all graduates of Ph.D. programs in mathematics need to acquire some teaching experience.

The CUPM Panel on College Teacher Preparation therefore recommends that graduate mathematics departments explore and develop new programs providing carefully supervised teaching experience for beginning teaching assistants, and that such teaching experience be regarded as an integral part of the Ph.D. program. We recommend that student teachers be regularly and effectively evaluated with regard to their achievements in this program. We further recommend that members of the graduate mathematics faculty be assigned responsibility for the teaching assistant program and that this responsibility constitute a recognized part of their teaching loads.

*According to a 1965 study, in more than half of the universities reporting, 40% or more of the freshman-sophomore teaching load is assigned to teaching assistants. (Conference Board of the Mathematical Sciences, Report of the Survey Committee, vol. I (1967), p. 36.)

ORIENTATION OF TEACHING ASSISTANTS

Richard C. DiPrima
Rensselaer Polytechnic Institute

These remarks about orientation, assistance, and supervision of teaching assistants deal with the conduct of our Calculus I course. This course is taken by approximately 750 students with a very high aptitude for mathematics and science. It is taught primarily in single sections of 25-30 students that meet four times a week. Two experienced professors are co-supervisors of the course. Approximately 14 teaching assistants teach individual sections; for a few it is their first classroom teaching experience. In addition to supervising the overall conduct of the courses, the two professors help the teaching assistant to develop good classroom teaching skills (insofar as these can be recognized). A conscious effort is made to allow each new teaching assistant to learn and to develop independently but, at the same time, to provide him with counsel and supervision.

During the week before the fall semester begins, there is a one-day orientation meeting which deals with (a) administrative matters and operating policies of the department and Institute, (b) procedures in teaching the assigned class and pedagogical techniques, and (c) course content. There are meetings at approximately two-week intervals of the teaching assistants and course supervisors as a group during the semester. At these meetings items such as material to be covered for the next few weeks, grades and grade distributions, and use of available audiovisual materials are discussed. In addition, each teaching assistant consults individually with one of the co-supervisors. Finally, the new teaching assistant attends a required course, College Teaching, which is conducted by the Graduate School and which meets once a week.

Perhaps of particular interest, and more distinctive in our program, is the special effort of the co-supervisors to familiarize the inexperienced teacher with the material of the course and what is expected of the student. Each teaching assistant receives an outline of the entire course showing the approximate amount of material to be covered in each lecture and indicating when tests are to be scheduled. This material is accompanied by:

(a) A copy of the final examination of the previous year and a set of answers and the grading of the examination. This is a great help in showing what is expected from the students and at what level the course should be taught.

(b) A typical set of reading and problem assignments for a period of eight lectures. The teaching assistant makes up his own assignments.

(c) A sample one-hour quiz. Again, the teaching assistant makes up his own tests and checks them, at least the first few, with a co-supervisor.

(d) A detailed set of comments about the course which indicates material to be emphasized, material to be skimmed, theory students are responsible for, and so on.

The set of comments described in (d) includes remarks on different sections such as "emphasize definition X"; "this material should be a review"; "assign problem X - it will be useful in section Y"; "we have a good transparency that illustrates theorem X"; "the discussion here is not easy but we do expect the student to know definition X"; "the emphasis here should be on the development of the properties of logarithm and exponential function"; and so on. The comments are more complete than we can illustrate here. They were carefully prepared and are regularly revised to take advantage of our experience. We have found the comments to be extremely helpful to the teaching assistants <u>and</u> to the faculty who are teaching the course.

In concluding these brief remarks we mention two items not directly related to orientation that may be of interest to the reader. We have conducted several different experiments in teaching classes of different sizes and combinations of lectures (60-100 students) and problem sessions (20 students). Also, several of us have made an effort to use overhead projectors and transparencies readily prepared in the department offices in lecture sections and/or regular sections and also to use other audiovisual material. In response to a questionnaire given to junior-senior students taking Advanced Calculus, Mathematical Analysis, Computing, or Operations Research, the students answered the questions:

In the freshman and sophomore courses would you have preferred .

(a) single sections only (30 students), many of which would be taught by graduate students <u>161</u>

(b) large lecture sessions (100 students) taught by experienced faculty members <u>18</u>

(c) combination of large lecture sessions and small problem sessions <u>106</u>

If an overhead projector was used in one of your basic classes how do you feel the use of the projector compared to the use of the blackboard?

(a) prefer projector <u>110</u>

(b) prefer blackboard <u>58</u>

(c) both about the same <u>51</u>

One student made the following comment about a professor whom he had in two consecutive semesters: "Last semester with an overhead projector you were excellent, this semester with a blackboard you are just good."

AN ORIENTATION PROGRAM FOR TEACHING ASSISTANTS

Siegfried K. Grosser
University of Minnesota

For the second Orientation Program for Teaching Assistants in mathematics conducted in September 1971 an effort was made to implement recommendations made the year before and to create a permanent, yet flexible, framework for the activities of future programs. The general purpose of these concentrated orientation and training sessions is to acquaint TAs with some basic teaching techniques and to give them increased awareness of the importance of their function in the educational enterprise. Specifically, the following goals were to be reached:

1. To improve the participants' teaching skill and effectiveness through demonstration, practice, criticism, and discussion.

2. To improve their familiarity with the fundamentals of the calculus.

3. To acquaint them with some teaching aids and their uses, notably computers and films (selected from the Calculus Film Series of the MAA).

4. To give them a more general perspective of the calculus within the framework of the natural sciences and thereby heighten their sensitivity for the need to motivate their future undergraduate students.

5. To provide them with such technical information as they will need in their dual capacity as teaching assistants and graduate students.

6. To provide a social setting in which the participants would get acquainted with each other, with other graduate students, and with the faculty.

Thirty-four hours, over an eight-day period, were allotted for the program in the fall of 1971, thirty-three in 1970; in each year, activities were concentrated in the morning hours so as to give the TA time in the afternoons to plan his own academic program. In 1970 the number of participants was 29, all of them first-year TAs, while only 18 of the 28 participants in the 1971 program were new to the department. The cost of the 1970 program was one month's salary for the director and one month's salary for his aide; in 1971 two aides assisted the director.

Naturally, an eight-day immersion program as described below, although concentrated, cannot create superb teachers overnight - although in individual cases significant improvement can be seen - and it cannot accomplish the task of acquainting the future TA with the whole range of intellectual, educational, and professional issues confronting the university; hence, although the program in its future form, twice amended, is likely to have maximum effectiveness in achieving its (limited) objectives, the wider task remains unfinished. For this reason a recommendation is being made that a follow-up Apprenticeship Program be created during which the new TA would teach in close collaboration with one or more faculty members and would continue to participate in a seminar devoted to matters of teaching. In addition, TAs are urged to attend the activities of the Academic Forum, a permanent series of weekly lectures and other events, parallel to the Colloquium, which grew out of last year's orientation program. A description of its goals and a schedule of events are given below.

The activities in the Orientation Program and in the Academic Forum can be grouped into five major categories as follows:

(a) Lectures in which the TAs are familiarized with their future teaching duties as well as lectures analyzing or demonstrating teaching techniques.

(b) Practice lectures (concerning topics from trigonometry, finite mathematics, and the calculus) given by the TAs themselves. These are followed by discussions.

(c) The showing of films and slide collections related to these topics.

(d) Lectures, films, and debates on topics of general interest such as computers, logic, history and philosophy of mathematics and science, new or unusual applications, the status of TAs, issues in higher education, etc.

(e) Social activities.

The activities listed under (a) and (b) constitute the backbone of the program; those labeled (c) are essential if instructional technology ever is to play its proper (supporting) role in the teaching of the subject; those under (d) reflect the need for a widening of intellectual perspectives and concerns without which genuine curricular innovation and, eventually, educational reform would be impossible.

Events labeled (a), (c), and (d) were scheduled in a large auditorium, so that interested parties - students, faculty, visitors - would have a chance to attend; those under (b) require a smaller room where closer rapport can be achieved and stage-fright minimized. For this purpose the group was split into halves, one directed by the organizer, the other by his two assistants, experienced TAs who were selected from over 100 TAs on the basis of their excellence in teaching. After each participant had given his first practice lecture, the directors exchanged groups. The division into two groups created sufficient time for each participant to deliver two practice lectures of fifteen to twenty minutes' duration. In the schedule given below these are designated as "L"; they comprise a review of the highlights of the calculus and some more elementary topics such as trigonometry. These lectures were to be given on an elementary level, with emphasis on clarity of expression and explanation rather than complete mathematical rigor, and equal emphasis on good lecturing habits. This catch-all phrase includes the numerous aspects of a teacher's classroom performance listed in standard evaluation questionnaires. For the first four lectures (L 1-4 in the schedule), no instructions or directives (as to teaching) were issued. Among the practice lectures video taped were those of the people who gave the first four. At the end of the program some of these video tapes were viewed and improvement between the individual's first and second lectures was noted. Each practice lecture was followed by a short critique. The participants were also shown examples of construction of a test and a course syllabus and themselves made tests and syllabi as an exercise. Before arrival they were asked to do some reading in three or four standard works on the history of mathematics and mathematical ideas. And since it has been found that graduate assistants, in tutorials and when teaching sections, may very well have trouble doing problems in lower-division mathematics, a proficiency test covering this material was added.

Participants were given the opportunity to learn about the writing of simple computer programs since some sections of the introductory calculus course at Minnesota are now using the approach to calculus via computers pioneered by W. Stenberg, _et al_. It was found that a majority of the participants already had some familiarity with computers. The discussion also touched on the use of computers for the purpose of introducing concepts such as the definite integral and convergence of sequences.

The exact schedule of the program was this: (L = lecture, F = film)

_Th 16_: Welcome Address (8:30-9:00); "Graduate Student - Enlarging Perspectives" (9:00-9:20); Proficiency Test (9:30-10:20); Coffee Hour (10:30-11:00); L 1-4; F 1,2 (11:00-1:00); Common Lunch (1:00-2:00); The Making of an Examination (2:10-3:00); Introduction to Computers I (3:10-4:00).

_Fr 17_: The Making of a Course Syllabus (8:30-10:00); F 3 (10:05-10:30); Coffee Hour (10:30-11:00); Slide Lecture (11:00-11:50); "Some Principles of Good Teaching" (12:00-12:50); Common Lunch (1:00-2:00); Demonstration of Teaching Techniques (2:10-3:00); L 5-7 (3:10-4:00).

_Tu 21_: Comments on Proficiency Test (10:00-10:30); L 8-11, F 4,5 (10:30-12:00); Introduction to Computers II (12:10-1:00); "Brain Express - Who Runs the Railroad?" (2:10-3:00); L 12-14 (3:10-4:00).

_We 22_: Examination and Syllabus: Critique (10:00-10:30); L 15,16; F 6-9; L 17-19 (10:30-1:00); "The Roles of Set Theory in Mathematics" (2:10-3:00); "The Mathematics Library" (3:10-3:30); "The Recognition and Evaluation of Teaching" (3:30-3:50).

_Th 23_: L 20-23; F 10-13 (8:30-10:30); Coffee Hour (10:30-11:00); L 24-28 (11:00-1:00); "Some Unusual Applications of the Calculus" (2:10-3:00); Introduction to Computers III (3:15-4:00).

_Fr 24_: L 29-31; F 14-16 (8:30-10:30); Coffee Hour (10:30-11:00); L 32 (11:00-11:30); Viewing of Video Tapes (11:30-12:15); Evaluation of Program (12:20-1:00).

Mathematics TAs were also expected to participate in two short college-level orientation programs; this is the reason for some of the irregularities in the timetable given here.

Systematic evaluation of the effectiveness of the program is based on (a) use of video tape, (b) questionnaires, (c) visitation by faculty, (d) student evaluation. As concerns (d), after last year's program, and for the first time ever, no student complaints concerning bad teaching were registered;

however, when the department hired two inexperienced TAs during the winter quarter, such complaints occurred again. Acceptance of the program by the participants was unequivocal; by a margin of 25-2 they voted in favor of a continuing program in matters of teaching and education.

The Academic Forum has the following objectives:

(a) The discussion of matters of interest to the mathematical community - students and faculty - other than specialized research; in particular, the expository presentation of new or unusual developments in the subject and the examination of some of its cultural and historical complexity.

(b) The establishment of a mode of interaction, other than the purely technical one, between the School of Mathematics and other departments and collegiate or administrative entities in the university.

(c) The creation of a dialogue concerning matters of teaching and education - undergraduate and graduate - and the discussion of significant trends and practices.

During the academic years 1970-71 and 1971-72 the following activities have been featured:

Lectures

- "Unsolved Problems in Mathematics"
- "Applications of Mathematics to Economics I & II"
- "A One-hour Introduction to Computers"
- "Computers and Calculus"
- "A Tale of Degeneracy and Decay (Symmetry Principles in Quantum Mechanics)"
- "Topological Chemistry"
- "Computers and Linear Algebra"
- "The Intellectual Challenges in the University"
- "Knots"
- "Accountability Comes to Minnesota"
- "A Program of Educational Development in Mathematics"
- "The History of Fourier Series"
- "Computer-generated Pictures for Teaching Calculus"
- "A Working Prof's Misgivings about Some Recent Trends in Undergraduate Education"

Films

- "John von Neumann"
- "Geometric Thinking"
- "Can You Hear the Shape of a Drum?"
- "Göttingen and New York - Reflections on a Life in Mathematics"

Debates

- "The Graduate Program in Mathematics I & II"

Finally, it should be mentioned that faculty and graduate student interest in these offerings - Orientation Program and Academic Forum - and in matters of teaching and education in general has noticeably increased. One reason for this seems to be the fact that in a large department, with a faculty close to 100, multifaceted programs such as the ones described here, which lend visibility to and provide a forum for the diverse faculty interests and competencies that do not surface in the traditional set-up, tend to have a unifying and stimulating effect. This, more than mere administrative encouragement and support, augurs well for the future.

LARGE-GROUP INSTRUCTION AT THE UNIVERSITY OF MARYLAND

David Schneider
University of Maryland

Math 110-111 is a precalculus and probability course which is intended for students majoring in the biological, social, and management sciences. An enrollment of over 3,000 students per semester justified a major commitment by the mathematics department and resulted in a course which is beyond the capabilities of the standard classroom situation.

Goal. Many of the students taking Math 110-111 have a mental block towards mathematics--a block caused not only by a feeling that mathematics is too abstruse for them to understand but also by a belief that it is not a living, growing discipline which can play a vital role outside of the physical sciences. The primary goal of Math 110 and 111 is to break through this block and educate the student in the ways that mathematicians approach problems. Throughout the course mathematical models are constructed and mathematical concepts are developed to answer questions about these models.

Format. Students meet in groups of about 25 for three hours per week with a graduate assistant. The first 30 minutes of the average class consists of a discussion of concepts and homework conducted by the graduate assistant. Then new material is presented via TV using a pre-taped lecture which is transmitted from the TV studio over a closed-circuit system. Printed lecture notes are distributed at each lecture, allowing the students to concentrate fully on the TV presentation.

TV Lectures. Extreme care went into the preparation of the TV lectures in an attempt to meet professional standards of TV production. Three faculty members of the mathematics department were assigned the making of a series of 63 video tapes as their sole duties for 1½ years. The Speech and Drama Department provided a professional director, a well-equipped studio, and a seven-man technical staff. On the average, the members of the mathematics department devoted about 75 man-hours to the making of each lecture. Although one person had the primary responsibility for each lecture, the lectures were thoroughly discussed by the other faculty members working on the course. Careful consideration was given to content, clarity, relevance, and style. Full use was made of the capabilities of the TV media by using slides, mats (subtitles), film clips, and imaginative props. As an example, part of a lecture on an application of mathematics to medicine was filmed at the National Institute of Health, with the commentary given by the director of a cancer research project.

Notes. Instead of dissipating their energies on (mis)copying information, the students concentrate on the TV lectures and rely on printed lecture notes which are distributed at each lecture. The lecture notes are the focal point of the learning process. Although students gain an insight into the concepts while listening to the TV presentation, it is through the second exposure of reading the lecture notes that the material is firmly absorbed. The lecture notes contain exercises which force the student to work with the new concepts and which raise questions to be resolved in future lectures.

Graduate Assistant. The sixty graduate assistants working with Math 110-111 are usually first-year graduate students. Course work makes substantial demands on their time, and therefore it is imperative that the time devoted to teaching be used effectively. The students taking Math 110-111 are often weak mathematically and need close personal attention. The graduate assistant, free from the responsibilities of preparing lectures and examinations, can devote his entire effort to meeting this need. As a result of working with the students in class, talking with them during office hours, and looking over homework and examinations, the graduate assistant is able to ascertain their difficulties. A set of daily comments distributed to the graduate assistant points out some places where difficulties could occur and suggests homework exercises to be carefully covered in class.

Response from Other Departments. "For several years I have been trying to show my advisees a relation between their required math course and zoology. Mathematical models are of increasing importance in all fields of biology and we find too many of our students shying away from even simple models. The early introduction (before calculus) of such models should improve their receptions in our zoology classes."

"As a teacher of statistics and mathematical psychology I am less concerned with students' prior exposure to specific mathematical concepts than I am with their ability to appreciate mathematics as a representation of structures. The approach taken in these notes seems to present this aspect of mathematics rather effectively."

"I have circulated your lecture notes among several members of our Department of Business Administration. The reaction has been extremely favorable. We are pleased with your use of practical examples and organization of the course."

Student Survey.

"The commitment made to Math 110 by the Mathematics Department has been substantial."

Highly agree	31%	Highly disagree	2%
Slightly agree	42%	Slightly disagree	5%
Neither agree nor disagree	19%		

"My original view of TV in the college classroom was favorable."

Highly agree	7%	Highly disagree	30%
Slightly agree	14%	Slightly disagree	31%
Neither disagree nor agree	18%		

"Regardless of my previous view, I feel that the use of TV with prepared lecture notes in Math 110 has been effective."

Highly agree	33%	Highly disagree	6%
Slightly agree	39%	Slightly disagree	11%
Neither disagree nor agree	10%		

Student Comment. "The TV, lecture notes, and classroom instructor serve as three different teachers of the course, and I find that is much more enjoyable than if one teacher taught all the time."

AN EXAM-TUTORIAL PROGRAM IN CALCULUS AT M.I.T.

Arthur P. Mattuck
Massachusetts Institute of Technology

Many freshmen enter college already knowing some calculus, and this causes difficulties in the freshman calculus course. Even those who know a lot sometimes do poorly since the material doesn't seem fresh, and as a result they cannot bring themselves to study it. Steering such students into "honors" courses does not help, since most of them are not theoretically inclined. To make matters worse, for the past three years at M.I.T. the freshmen have been graded Pass-Fail. As a result, many feel they know enough calculus to pass, do little studying, and end up walking into quicksand.

A program to deal with both these problems was tried last year and is continuing this year. It handles the 650 students enrolled in standard first-year calculus. All of these students get the same course (there is no honors course), the first semester of which corresponds roughly to the Advanced Placement high school course, somewhere between the AB and BC levels. The program has an examination-tutorial part and accompanying course options.

Exam-tutorials. Each semester's work is broken down into six approximately 2-week units. Five or more equivalent 40-minute examinations are prepared for each unit. A student may ask for an examination on any day by going to a special examination room. The room is open for two hours on each day. The proctor gives him at random one of the five examinations for that unit. It is graded immediately afterwards in an adjoining room by whatever tutor happens to be free; he sits with his answer book at a table with the student and discusses his paper with him. He may ask the student a leading question or two to clarify an answer. If a student claims he couldn't remember a formula, the tutor may give him the formula and see whether he can now do the problem. Or he may tell the student a mistake was made and see if the student can find it. He explains how to do the problems that were missed. To pass, the student must get 75-80 per cent and demonstrate some knowledge of each of the four problems. The pass or fail is recorded in a looseleaf book (one page per student), and his examination paper is filed. If a student does not want to stay for the grading, the paper is graded without him--but most stay.

If the student passes, he is ready for the next unit; if he fails, he takes another of the five versions whenever he feels ready. The process is repeated until he passes or until all versions of that exam have been taken. When a student has passed all six examinations, he has passed the course, there being no final. If he passes at least four examinations by the end of the term, he receives an Incomplete, which allows him to finish the work during January or early in the next semester on the same basis. Otherwise he fails, but in repeating the course he is not required to repeat the exams already passed.

A detailed syllabus is prepared for each unit: textbook reading, supplementary notes when needed, problems, and specimen examinations. An outline of this is sent to students during the summer, and many students without Advanced Placement cash in on their high school background by taking examinations during the week before fall classes start; the average student passes one examination.

Class Options. Since the examination procedure is self-paced, it is necessary to have classes which support this. First-semester calculus is offered therefore in three versions--A, B, C--which vary in starting point and pacing. All are aimed at the same examinations, however.

Option A starts from the beginning, covers the material of the six exams in 12 weeks, and is primarily for students with little high school calculus.

Option B starts with a review of the second examination, then proceeds at a normal pace with the rest. The course is finished in 9-10 weeks, the remaining time being used by the student for review or working on other courses.

Option C starts with the third examination and is paced to finish the course in six weeks, after which the student begins the second semester work. He will thus finish either in March, or at the end of January by taking an accelerated version during the 4-week January Intersession. Option C is for students either with a lot (perhaps a year) of high school calculus or who are quite bright. It was even taken by some with Advanced Placement who felt they were rusty from a summer's inactivity and needed a 6-week review. For option C the remaining six weeks of last April and May were filled by elective courses Vector Integral Calculus and Calculus Theory.

Students may enter whichever option they want. All are taught at the same hours, so they can switch from one section to another if it seems desirable. Last year very few students dropped back to a more slowly paced section, however, even when it became obvious that they were behind the rest of their classmates in passing examinations. Options A and C are taught in two lectures and two recitations per week; option B is taught in two 2-hour recitations per week to allow more flexibility in adapting to the needs of this particular group. About one third of the freshmen elected each option.

For those "A" students who find themselves far behind at midsemester, special review classes are run during the last four weeks, aimed just at getting these students through the examinations.

Comments. There was no attempt made at objective evaluation. Four times a year, however, the students write a few sentences describing how the course is going for them; questionnaires were also distributed at the end of the year.

For the students, the important and popular features of the program were the self-paced aspect which allowed them to take an examination when they (and not their teacher) felt ready, and the general supportive nature of the examinations. These were perceived as being primarily a teaching device, rather than a checking and punitive one. The conference with the tutor was liked, and tension of examination-taking was greatly reduced. No stigma is attached to failing since it is so common: the pass-fail rate is about 2-1, declining to 1-1 near the end of the term when students tend both to be poorer and to take more chances.

For the mathematics department, on the other hand, the most important feature is the guaranteed minimal performance--a student has to prove at each stage that he is competent before going on to the next.

Difficulties. Procrastination is an important problem. It is not serious the first semester: after six weeks the average student in each option was one examination behind (i.e., those in option C had taken five examinations, etc.). At the end of the term about ten failed and about 175 (little over one quarter, mostly in option A) received Incompletes, all but a dozen of which were made up early in the second term. Second semester brought a general slump in all subjects and heavy procrastination in taking examinations. We announced at the end of March that those who weren't finished by the end of the term would take the missing examinations (up to a maximum of three) on a one-shot basis at final-examination time. Alternatively, a regular final examination could be given to this delinquent group.

The links between recitation teacher and student are on a less artificial basis than usual; this hurts some of the teachers who see their classes dwindling as the students look elsewhere. During second semester, attendance at all recitations and lectures declines.

Students who fail two or three times are sometimes at a loss as to what to do next; it would be desirable to give them a sheet suggesting what to do for more practice.

Details. For the exam-tutorial program, one needs an examination room or rooms, a tutorial room with enough 2-man tables, and space for the students to wait in. (Occasionally they may have to wait as long as 15-20 minutes if there is an unexpected crush.) Tutors last year were graduate TAs, upperclass mathematics majors, and a few sophomores. Undergraduates were paid $2 an hour. (The seven sophomores were given a choice of academic credit or money; two decided to take the cash and let the credit go.) The TAs this year act as head tutors, serving as general supervisors and as referees if trouble arises. They also do some tutoring and conduct the staff meetings. A proctor and record keeper are needed. It is necessary to allow about 10 minutes per student per tutor (15 minutes toward the end of the term when students are poorer, worse prepared, and are hoping for more explanation). The average student takes 10 examinations per semester. The total cost of the exam-tutorial program would thus be about $5 or $6 per student per semester if only undergraduates are used as tutors. Except for a peak period just before the end of the term, demand on the tutors is rather uniformly distributed because of the A, B, C options and averaging due to local procrastination. Tutors should have weekly staff meetings in order to discuss tutoring difficulties and to change problems on examinations where it seems desirable. The examinations were mostly prepared by a graduate TA during the previous summer, under faculty direction.

The teaching requires a large number of recitation rooms available at the same hours (we use Tues-Thurs 11-2, a generally unpopular hour with the other departments). The method can be used with standard grading systems as well, the grades being tied to the number of examinations passed and (perhaps) to the number of tries needed for each one.

The method was freely adapted from a self-paced study course in freshman physics, without classes, developed on an experimental basis by Dr. Ben Green of the Educational Research Center at the Massachusetts Institute of Technology, which in turn was based on a similar program in freshman psychology developed by Professor F. Keller at Arizona State University. It requires careful planning both in the logistics and the syllabus. Since the passing grade is high, the examinations must seem fair, be individually well-balanced, cover collectively the entire syllabus, and represent a reasonable expectation of the students at the given school. Designing the examinations forces the faculty member to decide what he really feels the student must be able to do, a very sobering experience in these days when theory holds sway.

A NEW APPROACH TO MASS INSTRUCTION

John W. Riner
The Ohio State University

The Mathematics Department at The Ohio State University is in the process of changing the instructional techniques used in teaching its three large-enrollment elementary course sequences. The three sequences all contain calculus with varying applications. The depth of understanding expected of the students varies with the sequence. The students are our large middle group not in honors courses and not in remedial courses.

The program being developed (under the name of CRIMEL, Curriculum Revision and Instruction in Mathematics at the Elementary Level) is intended to provide each student enrolling in one of the sequences the opportunity to start his training in mathematics at an appropriate level and to proceed at a pace compatible with his ability, background, and interest. The ultimate goal of the program is to provide individualized instruction for each student enrolled in any of our elementary courses.

At the present time, certain 5-quarter-hour courses are being decomposed into shorter units (mini-courses carrying one, two, or three hours of credit) from which can be constructed any of the existing course sequences. Our first step was taken during the Autumn Quarter, 1970, when each of three beginning courses was decomposed into two mini-courses, one for three hours of credit, the other for two hours of credit. All students, approximately 4200, enrolled in any one of the courses took the same beginning mini-course. After two weeks of instruction over the same material (algebra), a test was given to all students. On the basis of the results of this test, students were advised to continue their course work at the traditional pace, or to add a 3-hour mini-course and to continue their study at an accelerated pace (earning eight hours of mathematics credit during the quarter), or to reduce their mathematics load to the 3-hour course only. Roughly five per cent of the students were in the accelerated pace treatment, 65 per cent were at the regular pace, and 30 per cent were at the reduced pace.

This first effort is viewed as successful. As might be expected, it was especially effective for the students who either accelerated or reduced the pace of their study in the courses. A questionnaire completed by 2800 students at the end of the quarter showed that 55 per cent of those responding thought that the program should be continued, 23 per cent thought it should not, and 23 per cent had no opinion. For accelerated students these percentages were 65, 7, and 28 and for reduced-paced students they were 64, 16, and 20. The regular-paced students for whom the treatment was more conventional responded with percentages of 48, 27, and 25. Another indicator of the success of the initial effort is the number of students who dropped the courses under the new techniques. Only 150 out of 4200 students dropped, as opposed to 350 out of 4000 who dropped from the same courses in the fall of 1969.

The basic unit of the future system will be the instructional module consisting of the mini-course and a collection of instructional aids (including instructors) that will enable the student to proceed through the material of the course in a way best for him and at his own pace. Our efforts to provide a variety of techniques and materials to supplement the work of our instructors are proceeding well. We have developed in preliminary form a set of 12 video tapes, each of about 15 minutes' duration, for viewing both in and outside of class. We are experimenting with computer-assisted instruction. Our capability to provide tests on the same material at frequent intervals is developing well. Our tutoring capability is being upgraded rapidly. These activities involve and have involved seven of our regular faculty members as well as a sizable staff of supporting personnel.

One of the greatest problems encountered is that of managing such a system in a manner that is effective for large numbers of students. The need for rapid and accurate information flow and the need for better record-keeping techniques became very clear during our initial operation.

To facilitate the record-keeping and to give the students a chance to identify better with a "class," we have changed our system design for the 1971 academic year. We are using the concept of the "cluster." A cluster consists of about 150 students, a cluster leader who coordinates the instruction, four to six instructors, and a clerk-tutor. These persons operate in a group of six classrooms, four of which are equipped with closed-circuit TV monitors. A testing center exists where students can be tested over certain blocks of material with a great deal of time flexibility.

As students are able to move ahead, as they satisfy both themselves and us in their performance on tests, they are grouped with others in their cluster moving at about the same pace and are provided appropriate guidance and instruction. Thus the flexible pacing is accomplished within the cluster rather than in the large body of all the students. Because of better ability to maintain records, the flexible testing program is moving forward rapidly. The testing of certain topics can be done over a relatively wide time interval--three to four weeks. This allows retesting not only for students who fail tests but also for students who feel that a given test score does not reflect accurately their knowledge of a topic. Students may request a test whenever they are ready for it.

In a pilot project for the cluster concept (Spring Quarter, 1971) the students responding to our survey supported the flexible testing program strongly. Ninety-five per cent viewed it favorably and as having value. In this same pilot project 63 per cent of the students favored continuing the project, 20 per cent felt it should not be continued, and the rest gave no opinion.

As indicated in this report, we are making progress in solving the basic operational problem of implementing our program for large numbers of students. We are making some progress in developing the teaching aids we hope to use in individualizing instruction. We have a group of faculty members interested in the various facets of the problem. We are moving ahead cautiously, evaluating the effect of our changes, and adjusting our operation in light of the evaluation.

In addition to the basic problems, we are aware of several more subtle problems that require attention. One of these is induced by the role change required by our instructors. They do not have classes in the usual sense for whole terms. Instead they have changing or different groups of students for much shorter periods of time. They lose the big side payment of seeing "their classes" grow. They must function as members of teams. These problems require at least a careful orientation program for the persons who will work in the program.

Another problem we face is that of getting the student to use the program effectively. The student has more responsibility for his own progress than under traditional treatment. How do we make sure that he assumes this responsibility?

The department has a very real and deep commitment to the program. Our task is to provide good instruction in mathematics to a large group of individuals. We feel that we must take their individuality into account when we perform this task.

THE CALCULUS EXPERIMENT AT U.C.S.D.

Burton Rodin
University of California, San Diego

In recent years a number of new teaching programs have been developed in colleges across the country. It is remarkable that these various programs, which have evolved independently in response to common problems, share the same basic features. These common features are (1) subdividing a course into small units, (2) encouraging mastery of one unit before offering the next unit, (3) tolerating some amount of self-pacing by students, and (4) allowing some method for students to make up a failed examination without excessive penalties. This convergence of solutions may point to an idea whose time has come.

The program evolved at U.C.S.D. is more extreme than the others. The result is that advantages are maximized along with disadvantages.

An idealized description of the program. We* began with the belief that our goals for the freshman calculus course could be made explicit. This meant that we ought to be able to make a list of all the concepts and techniques that we expected an "A" student to master. The next task was to prepare a list of basic problems which would test students and show whether they had mastered those goals. To illustrate: for the goal "To apply the Chain Rule as a technique in differentiating" we had the problem "Differentiate the following functions ..."; for the goal "To understand the geometric interpretation of the derivative" we had the problem "Figure (a) shows the graph of a function f; which of the figures (b), (c), (d), or (e) shows the graph of f'?". For our one-year course we ended up with about 100 goals and 100 basic problems.

Individual taste plays a role in compiling the list of goals. Whether or not mastery of the ε-δ definitions should be a goal was debated heatedly. At any rate, once the lists are made is there any reason to keep them secret from the students? Definitely not, we felt.

The final step was to prepare variations of the basic problems for use in official testing. Corresponding to basic problem number n we also had problem n', problem n'', etc. These variations were sometimes trivial to prepare--in the illustrations above we only needed to change the particular functions involved. In other cases the preparation of suitable variations required much thought.

Ideally, the course was to proceed as follows. Whenever a student felt he had mastered goal n (he could test himself with the nonsecret problem n), we allowed him to attempt problem n' as a quiz. If he failed to solve problem n', there was no penalty. If he wished to try again at a later time, we offered him problem n'', and so on. If at some time during the course he solved

*The reference is to a group of instructors at U.C.S.D., although the original idea of the program here is due to Professor John Evans.

a variation of problem n, then he was credited with mastery of goal number n. Any student who was credited with mastery of all the goals by the end of the course received a grade of "A".

Is it correct to assume that a student has mastered goal number n just because he managed to solve problem n""? After all, a skeptic would say, he did fail problems n', n", and n'''. Our answer is that this objection is merely a constraint on us; we must be sufficiently clever in composing problem n"" so that we can guarantee an affirmative answer to the question above.

Actual course administration. The course consisted of 500 students. There were three large lectures each week and two recitation meetings in smaller groups. During a typical week about four of the basic goals would be covered in the course of the lectures. The basic problems corresponding to these goals would be offered as homework and discussed during a recitation meeting. One recitation period a week was used for students to take quizzes (current problems as well as variations of past ones). There was no final examination--the time normally scheduled for that was used instead for additional quiz sessions. Due to popular request several additional quiz sessions were arranged each semester, usually in the evening.

Evaluation. Forty per cent of the students received a grade of "A" (that means mastery of all the goals). To say that their response has been enthusiastic would be an understatement. On questionnaires, students unanimously claimed to have learned more than they would have under the ordinary system, to have enjoyed the learning process more than in previous courses, and to have derived very general benefits in the area of "how to study."

The system is also attractive because of its adherence to basic principles of psychological learning theory. Aside from this and the overwhelming encouragement from students, the mathematics instructors judged the experiment successful on other grounds: compared with student performance during previous years, many more students had succeeded in working difficult mathematical problems. Even though it may have taken them several attempts to do it, final accomplishment itself demonstrates a mastery of ideas that only very few students had attained under the old system.

The main disadvantage of the system was the excessive man-hours required to administer it. Office hours that had been adequate in the past were no longer so; the hordes of previously apathetic students had been awakened to the possibility that mathematics courses could be mastered, and they were demanding help. Instructors were also heavily burdened by the necessity to make up countless variations of the basic problems week after week and to administer them. Several professors have testified that they spent twice as much time in administering the course as they spent in preparing and giving lectures. TAs also complained bitterly about the long hours they spent. Currently we are experimenting with new ways to administer the quizzes and office hours more efficiently.

Concluding remarks. Several modifications of the program have been tried out, e.g., severely limiting the number of make-ups allowed or adding ordinary examinations to the system. None of the modifications was very successful. The reason seems to be that in the pure form of the program students really feel that by studying hard they can get an "A". This feeling seems to be at the core of their enthusiasm. Examinations that could not be retaken, or similar modifications, destroyed that feeling, and many students lapsed back to their old apathy.

A key feature of this program is its adaptability. Often initial objections vanish when the flexibility is exploited. For example, there is the objection that this program does not require students to review material as for a final examination; instead, once they have mastered a goal they are free to forget it. One way to correct this defect would be to add review problems to the list of basic problems. There can be several basic problems which correspond to one goal; these basic problems might appear at intervals throughout the year.

Another objection is that the problem numbers themselves tell students what method is needed for the solution; the element of judgment is therefore not being tested sufficiently. This can be overcome by having several parts within one problem ("Evaluate the following integrals ..." would be one question; within it are integrals requiring the methods of partial fractions, integration by parts, etc.).

Special emphasis on goals such as theorem-proving, applications of concepts, etc., can all be tested. The program is limited only by the ingenuity of the person preparing the basic problems and their variations.

TEACHING ELEMENTARY MATHEMATICS TO LARGE GROUPS OF NON-MATHEMATICS MAJORS

A. Bruce Clark
Western Michigan University

Mathematics departments at many large institutions are faced with the problem of how to utilize regular faculty members most efficiently in teaching large numbers of students in elementary service or regular courses (calculus and below) for non-mathematics majors. One method is to offer such courses in many small sections taught by graduate assistants, usually inexperienced, and to use regular staff members in an administrative and training capacity to direct these assistants. The record of success (or of economy) for this method is not particularly impressive, and few faculty members have either the inclination or the ability to act as such academic administrators. On the other hand, since most of the content of such courses involves the perfection of elementary skills and the driving home of a few important ideas, they are not particularly suited to large-lecture operations taught directly by the faculty members.

Over the past two years Western Michigan University has experimented with another method of handling these courses. This system was developed with the advice of behavioral psychologists on the university faculty and has proved almost surprisingly effective.

Details of Operation

Under the system a senior faculty member is assigned a large class of this type (possibly up to 300 students) as half of his regular teaching load. He is given a graduate assistant to act as administrative assistant and also a number of undergraduate paper graders. A 4-semester-hour course operates as follows:

(a) The content is divided into 14 or 15 units, each of one week's duration. Prior to the start of each unit, students are given a rather precise statement of the objectives of the unit together with suggested textbook readings and suggested exercises. They are expected to prepare on their own before the unit begins.

(b) On Monday, before the first class of the unit, students may submit questions in writing and the professor spends the first 10 minutes of class answering the most relevant questions so that simple misunderstandings are rectified. Then a 40-minute test is given. This is graded the same day by the corps of paper graders. Each paper is graded P (Pass), M (Marginal), or F (Fail). A student who receives a P is given full credit for the unit and is exempted from attendance for the rest of the week.

(c) On Tuesday there is a formal large lecture in which the main points of the unit are discussed in expository fashion.

(d) On Wednesday another test is given on the material of the unit; any student who scores a P is exempt for the balance of the week.

(e) On Thursday the lecture is devoted to problem solving and trouble-shooting. The group now consists only of the weaker students, and the difficulties that have shown up on the two previous tests are analyzed in detail. Numerous office hours and opportunities for small group sessions are offered as well.

(f) On Friday still another test is given.

(g) Saturday is reserved for make-up tests.

(h) No final examination is given. Scores for each unit are determined from the results of the three tests by the following table:

P, -, - = 10	M, F, P = 7	M, F, M = 4
M, P, - = 10	M, M, M = 6	M, M, F = 2
M, M, P = 10	F, F, P = 6	M, F, F = 0
F, P, - = 8	F, M, M = 5	F, M, F = 0
F, M, P = 8	F, F, M = 5	F, F, F = 0

Grades for a 15-unit semester are determined by totaling unit scores and using the following formula:

141-150	= A	(94 - 100%)
131-140	= B	(87 - 93%)
119-130	= C	(79 - 86%)
109-118	= D	(73 - 78%)
0-108	= E	(0 - 72%)

These formulas for computing grades are announced to the students at the beginning of the course.

Conclusions

Such a mechanical system of conducting a mathematics course, in which students are rewarded for doing well by having to spend less time on the subject, runs against the instincts of many mathematicians. On the other hand, it turns out that this system works very well for students in service courses who look upon the material strictly as a tool subject.

The following points should be noted:

1. While mathematicians usually prefer to spend their class time lecturing on all the course material, it is agreed that a system which trains and encourages students to dig things out for themselves from books is preferable.

2. Students like knowing exactly where they stand in the course at all times. In particular, the opportunity later in the week to make up poor or marginal work with little or no penalty encourages students to work.

3. The grading scale listed here is stringent, yet the encouragement to students to work hard through the week to improve their grades on each unit has resulted in honest high grades. Typically, classes run to over 50% A's and B's and less than 10% D's and E's.

4. Students who take a course under this method and then go on to take further mathematics courses seem to do just as well as students coming from a more traditionally taught section.

5. Four large classes have been given at this institution under this system. At the end of the courses students filled out detailed questionnaires. Students in these courses rated their course higher in all standard evaluation categories than did students in regular elementary courses. Of most interest, however, were the parts of the questionnaires devoted to comments. Typical comments given by large groups of students are the following:

   (a) Given the choice, I would choose this course over a traditionally-structured course. (80%)

(b) I would expect to work less in a traditional course. (60%)

(c) The method forced me to work harder than I would have done normally, and I feel that I learned more.

(d) I like the idea of being able to relax if I make an extra effort early in the week.

Our Department intends to continue to work with this system. More detailed information can be obtained by writing to Professor Anthony Gioia, Department of Mathematics, Western Michigan University, Kalamazoo, Michigan 49001.

Publications of the Committee on the Undergraduate Program in Mathematics

GENERAL

Commentary on a General Curriculum in Mathematics for Colleges (1972)
CUPM Basic Library List (1965)
A Course in Basic Mathematics for Colleges (1971)

ON TEACHING MATHEMATICS

Qualifications for a College Faculty in Mathematics (1967)
A Beginning Graduate Program in Mathematics for Prospective Teachers of Undergraduates (1969)
Recommendations on Course Content for the Training of Teachers of Mathematics (1971)
Suggestions on the Teaching of College Mathematics (1972)

PREGRADUATE TRAINING IN MATHEMATICS

Pregraduate Preparation for Research Mathematicians (1963)
Preparation for Graduate Study in Mathematics (1965)

TWO YEAR COLLEGES

A Transfer Curriculum in Mathematics for Two Year Colleges (1969)
Qualifications for Teaching University Parallel Mathematics Courses in Two Year Colleges (1969)
A Basic Library List for Two Year Colleges (1971)

APPLIED MATHEMATICS AND COMPUTING

Tentative Recommendations for the Undergraduate Mathematics Program of Students in the Biological, Management, and Social Sciences (1964)
A Curriculum in Applied Mathematics (1966)
Mathematical Engineering: A Five Year Program (1966)
Recommendations on the Undergraduate Mathematics Program for Engineers and Physicists (1967)
Recommendations for the Undergraduate Mathematics Program for Students in the Life Sciences (1970)
Recommendations for an Undergraduate Program in Computational Mathematics (1971)
Preparation for Graduate Study in Statistics (1971)
Applied Mathematics in the Undergraduate Curriculum (1972)
The Introductory Statistics Course (tentative, 1972)

NEWSLETTERS

Number 5, September, 1970. Maintaining Mathematical Momentum
Number 6, December, 1970. The Beginning Teacher of College Mathematics
Number 7, February, 1972. New Methods for Teaching Elementary Courses and for the Orientation of Teaching Assistants

The Committee on the Undergraduate Program in Mathematics is a committee of the Mathematical Association of America charged with making recommendations for the improvement of college and university mathematics curricula. Financial support for CUPM has been provided by the National Science Foundation.

Additional copies of this NEWSLETTER may be obtained without charge from CUPM Central Office, P.O. Box 1024, Berkeley, California 94701.